**COURSE DESCRIPTION:**

COURSE: Geometry  
FOR: 9/10  
CREDIT: 1  
PREREQUISITES: Successful completion of Math 8 Honors or Algebra 1 and teacher recommendation

This course is designed to introduce the student to the concept of a mathematical proof, provide an understanding of different logical reasoning, and provide a working knowledge of geometrically related vocabulary, theorems, postulates, calculations and their application to practical problems. Also included are introductions to statistics, transformations, circles, and right triangle trigonometry. A thorough understanding of fractions, percentages and decimals is required.

**COURSE SYLLABUS:**

<table>
<thead>
<tr>
<th>Semester I</th>
<th>Chapter</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Foundations for Geometry</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Geometric Reasoning (Only the Vertical Angles Theorem in 12.7)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Parallel and Perpendicular Lines</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Triangle Congruence</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Properties and Attributes of Triangles (Omit 5.2 and 5.6)</td>
<td></td>
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<tr>
<td>6</td>
<td>Polygons and Quadrilaterals</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Semester II</th>
<th>Chapter</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Similarity</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Right Triangles and Trigonometry</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Extending Transformational Geometry</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Extending Perimeter, Circumference, and Area (omit 10.4)</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Spatial Reasoning (including 3-D Representations, Lateral and Surface Area of 3-D Figures) (See attached information!!)</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Circles (omit 12.4, 12.5, and 12.6)</td>
<td></td>
</tr>
</tbody>
</table>

**KEY COMPONENTS TO TESTING OUT**

1. Name of Course: Geometry
2. Course description (above)
3. Course syllabus (above)
4. Final Requirements (check those that apply)
   _X__ exam
   ___portfolio
   ___demonstration performances
   ___presentation
   ___papers
   ___projects
5. A description of the requirement(s) checked above and how it (they) will be assessed.

   The testing out exam will consist of 100 multiple choice questions with a value of one point each. Students may use a calculator but one is not necessary.

6. Grade calculation for attainment of C+

   To test out of Geometry, a score of at least 77/100, (77%, C+) must be attained.
11A Representations of Three-Dimensional Figures

Isometric drawing is a way to show three sides of a figure from a corner view. You can use isometric dot paper to make an isometric drawing. This paper has diagonal rows of dots that are equally spaced in a repeating triangular pattern.

**Example 2** Drawing an Isometric View of an Object

Draw an isometric view of the given object. Assume there are no hidden cubes.

In a perspective drawing, nonvertical parallel lines are drawn so that they meet at a point called a vanishing point. Vanishing points are located on a horizontal line called the horizon. A one-point perspective drawing contains one vanishing point. A two-point perspective drawing contains two vanishing points.

**Student to Student**

*Perspective Drawing*

When making a perspective drawing, it helps me to remember that all vertical lines on the object will be vertical in the drawing.

Jacob Martin
MacArthur High School
EXAMPLE 4  Relating Different Representations of an Object

Determine whether each drawing represents the given object. Assume there are no hidden cubes.

A

Yes: the drawing is a one-point perspective view of the object.

B

No: the figure in the drawing is made up of four cubes, and the object is made up of only three cubes.

C

No: the cubes that share a face in the object do not share a face in the drawing.

THINK AND DISCUSS

1. Describe the six orthographic views of a cube.
2. In a perspective drawing, are all parallel lines drawn so that they meet at a vanishing point? Why or why not?
3. GET ORGANIZED  Copy and complete the graphic organizer.

Know it! Note

<table>
<thead>
<tr>
<th>Type of Drawing</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>isometric</td>
<td>Corner view</td>
</tr>
<tr>
<td>orthographic</td>
<td>Top, bottom, front, back, left, and right views</td>
</tr>
<tr>
<td>perspective</td>
<td>Parallel lines are drawn so that they meet at vanishing point(s).</td>
</tr>
</tbody>
</table>
11A Homework Day 1

Draw an isometric view of each object. Assume there are no hidden cubes.

1. 

2. 

3. 

4. 
Determine whether each drawing represents the object. Assume there are no hidden cubes.

5. Object:

   ![Object Image]

   Drawing:

   ![Drawing Image]

6.

Follow the steps to complete the drawing of a triangular prism in one-point perspective.

a. Draw a dashed line from each vertex of the triangle to the vanishing point (point V).

b. Use the dashed lines as guides to draw a triangle with sides parallel to the first triangle.

c. Connect corresponding vertices of the two triangles. Use dashed lines for all hidden edges.

7. Draw a rectangular prism in a) 1-point perspective and b) 2-point perspective.

   a) ![Diagram a]

   b) ![Diagram b]

8. Is the picture in one-point or two-point perspective?

   a) ![Picture a]

   b) ![Picture b]
Surface Area of Prisms and Cylinders

Objectives
Learn and apply the formula for the surface area of a prism.
Learn and apply the formula for the surface area of a cylinder.

Vocabulary
lateral face
lateral edge
right prism
oblique prism
altitude
surface area
lateral surface
axis of a cylinder
eright cylinder
oblique cylinder

Why learn this?
The surface area of ice affects how fast it will melt. If the surface exposed to the air is increased, the ice will melt faster. (See Example 5.)

Prisms and cylinders have 2 congruent parallel bases. A lateral face is not a base. The edges of the base are called base edges. A lateral edge is not an edge of a base. The lateral faces of a right prism are all rectangles. An oblique prism has at least one nonrectangular lateral face.

An altitude of a prism or cylinder is a perpendicular segment joining the planes of the bases. The height of a three-dimensional figure is the length of an altitude.

Surface area is the total area of all faces and curved surfaces of a three-dimensional figure. The lateral area of a prism is the sum of the areas of the lateral faces.

The net of a right prism can be drawn so that the lateral faces form a rectangle with the same height as the prism. The base of the rectangle is equal to the perimeter of the base of the prism.

Lateral Area and Surface Area of Right Prisms

The lateral area of a right prism with base perimeter \( P \) and height \( h \) is \( L = Ph \).

The surface area of a right prism with lateral area \( L \) and base area \( B \) is \( S = L + 2B \), or \( S = Ph + 2B \).

The surface area of a cube with edge length \( s \) is \( S = 6s^2 \).

The surface area of a right rectangular prism with length \( l \), width \( w \), and height \( h \) can be written as \( S = 2lw + 2wh + 2lh \).
Finding Lateral Areas and Surface Areas of Prisms

Find the lateral area and surface area of each right prism. Round to the nearest tenth, if necessary.

A. the rectangular prism

\[ L = Ph \]
\[ = (28)(12) = 336 \text{ cm}^2 \]
\[ S = Ph + 2B \]
\[ = 336 + 2(6)(8) \]
\[ = 432 \text{ cm}^2 \]

B. the regular hexagonal prism

\[ L = Ph \]
\[ = 36(10) = 360 \text{ m}^2 \]
\[ S = Ph + 2B \]
\[ = 360 + 2 \left(54\sqrt{3}\right) \approx 547.1 \text{ m}^2 \]

The base area is \[ B = \frac{3}{2}aP = 54\sqrt{3} \text{ m} \].

1. Find the lateral area and surface area of a cube with edge length 8 cm. \[ L = 256 \text{ cm}^2; \ S = 384 \text{ cm}^2 \]

The lateral surface of a cylinder is the curved surface that connects the two bases. The axis of a cylinder is the segment with endpoints at the centers of the bases. The axis of a right cylinder is perpendicular to its bases. The axis of an oblique cylinder is not perpendicular to its bases. The altitude of a right cylinder is the same length as the axis.

Lateral Area and Surface Area of Right Cylinders

The lateral area of a right cylinder with radius \( r \) and height \( h \) is \( L = 2\pi rh \).

The surface area of a right cylinder with lateral area \( L \) and base area \( B \) is \( S = L + 2B \), or \( S = 2\pi rh + 2\pi r^2 \).
**Example 2**

**Finding Lateral Areas and Surface Areas of Right Cylinders**

Find the lateral area and surface area of each right cylinder. Give your answers in terms of \( \pi \).

A cylinder with a radius of 1 m and a height equal to 3 times the radius.

\[
L = 2\pi rh = 2\pi (1)(5) = 10\pi \text{ m}^2 \\
S = L + 2\pi r^2 = 10\pi + 2\pi (1)^2 = 12\pi \text{ m}^2
\]

**B** a cylinder with a circumference of \( 10\pi \) cm and a height equal to 3 times the radius.

Step 1 Use the circumference to find the radius.

\[
10\pi = 2\pi r \\
r = \frac{10\pi}{2\pi} = 5 \\
\text{Circumference of a circle}
\]

Step 2 Use the radius to find the lateral area and surface area.

The height is 3 times the radius, or 15 cm.

\[
L = 2\pi rh = 2\pi (5)(15) = 150\pi \text{ cm}^2 \\
S = 2\pi rh + 2\pi r^2 = 150\pi + 2\pi (5)^2 = 200\pi \text{ cm}^2
\]

2. Find the lateral area and surface area of a cylinder with a base area of 49\( \pi \) and a height that is 2 times the radius.

\[
L = 196\pi \text{ in}^2; \quad S = 294\pi \text{ in}^2
\]

**Example 3**

**Finding Surface Areas of Composite Three-Dimensional Figures**

Find the surface area of the composite figure. Round to the nearest tenth.

The surface area of the right rectangular prism is

\[
S = Ph + 2B \\
= 80(20) + 2(24)(16) = 2368 \text{ ft}^2
\]

A right cylinder is removed from the rectangular prism.

The lateral area is \( L = 2\pi rh = 2\pi (4)(20) = 160\pi \text{ ft}^2 \).

The area of each base is \( B = \pi r^2 = \pi (4)^2 = 16\pi \text{ ft}^2 \).

The surface area of the composite figure is the sum of the areas of all surfaces on the exterior of the figure.

\[
S = \text{prism surface area} + \text{(cylinder lateral area)} - \text{(cylinder base area)} \\
= 2368 + 160\pi - 2(16\pi) \\
= 2368 + 128\pi \approx 2770.1 \text{ ft}^2
\]

3. Find the surface area of the composite figure. Round to the nearest tenth. \( 239.7 \text{ cm}^2 \)
**Example 4**

**Exploring Effects of Changing Dimensions**

The length, width, and height of the right rectangular prism are doubled. Describe the effect on the surface area.

- **Original dimensions:**
  \[ S = Ph + 2B = 16(3) + 2(6)(2) = 72 \text{ in}^2 \]

- **Length, width, and height doubled:**
  \[ S = Ph + 2B = 32(6) + 2(12)(4) = 288 \text{ in}^2 \]

Notice that 288 = 4(72). If the length, width, and height are doubled, the surface area is multiplied by 2², or 4.

**Check It Out!**

4. The height and diameter of the cylinder are multiplied by \( \frac{1}{2} \). Describe the effect on the surface area.

   The surface area is multiplied by \( \frac{1}{4} \).

**Example 5**

**Chemistry Application**

If two pieces of ice have the same volume, the one with the greater surface area will melt faster because more of it is exposed to the air. One piece of ice shown is a rectangular prism, and the other is half a cylinder. Given that the volumes are approximately equal, which will melt faster?

- **Rectangular prism:**
  \[ S = Ph + 2B = 12(3) + 2(8) = 52 \text{ cm}^2 \]

- **Half cylinder:**
  \[ S = \pi rh + \pi r^2 + 2rh = \pi(4)(1) + \pi(4)^2 + 8(1) \\
  = 20\pi + 8 \approx 70.8 \text{ cm}^2 \]

The half cylinder of ice will melt faster.

**Check It Out!**

Use the information above to answer the following.

5. A piece of ice shaped like a 5 cm by 5 cm by 1 cm rectangular prism has approximately the same volume as the pieces above. Compare the surface areas. Which will melt faster?

---

5. The 5 cm by 5 cm by 1 cm prism has a surface area of 70 cm², which is greater than the 2 cm by 3 cm by 4 cm prism and about the same as the half cylinder. It will melt at about the same rate as the half cylinder.

**Think and Discuss**

1. Explain how to find the surface area of a cylinder if you know the lateral area and the radius of the base.

2. Describe the difference between an oblique prism and a right prism.

3. GET ORGANIZED Copy and complete the graphic organizer. Write the formulas in each box.
PRACTICE AND PROBLEM SOLVING

Find the lateral area and surface area of each right prism. Round to the nearest tenth, if necessary.

13. \hspace{2cm} 14.

15. a right equilateral triangular prism with base edge length 8 ft and height 14 ft

Find the lateral area and surface area of each right cylinder. Give your answers in terms of \( \pi \).

16. \hspace{2cm} 17.

18. a cylinder with base circumference \( 16\pi \text{ yd}^2 \) and a height equal to 3 times the radius

Multi-Step Find the surface area of each composite figure. Round to the nearest tenth.

19. \hspace{2cm} 20.

Describe the effect of each change on the surface area of the given figure.

21. The dimensions are tripled.

22. The dimensions are doubled.

23. Biology Plant cells are shaped approximately like a right rectangular prism. Each cell absorbs oxygen and nutrients through its surface. Which cell can be expected to absorb at a greater rate? (Hint: 1 \( \mu \text{m} = 1 \text{ micrometer} = 0.000001 \text{ meter} \)
24. Find the height of a right cylinder with surface area $160\pi \text{ ft}^2$ and radius $5 \text{ ft}$.

25. Find the height of a right rectangular prism with surface area $286 \text{ m}^2$, length $10 \text{ m}$, and width $8 \text{ m}$.

26. Find the height of a right regular hexagonal prism with lateral area $1368 \text{ m}^2$ and base edge length $12 \text{ m}$.

27. Find the surface area of the right triangular prism with vertices at $(0, 0, 0)$, $(5, 0, 0)$, $(0, 2, 0)$, $(0, 0, 9)$, $(5, 0, 9)$, and $(0, 2, 9)$.

The dimensions of various coins are given in the table. Find the surface area of each coin. Round to the nearest hundredth.

<table>
<thead>
<tr>
<th>Coin</th>
<th>Diameter (mm)</th>
<th>Thickness (mm)</th>
<th>Surface Area (mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penny</td>
<td>19.05</td>
<td>1.55</td>
<td></td>
</tr>
<tr>
<td>Nickel</td>
<td>21.21</td>
<td>1.95</td>
<td></td>
</tr>
<tr>
<td>Dime</td>
<td>17.91</td>
<td>1.35</td>
<td></td>
</tr>
<tr>
<td>Quarter</td>
<td>24.26</td>
<td>1.75</td>
<td></td>
</tr>
</tbody>
</table>

32. How can the edge lengths of a rectangular prism be changed so that the surface area is multiplied by $9$?

33. How can the radius and height of a cylinder be changed so that the surface area is multiplied by $\frac{1}{4}$?

34. **Landscaping** Ingrid is building a shelter to protect her plants from freezing. She is planning to stretch plastic sheeting over the top and the ends of a frame. Which of the frames shown will require more plastic?

35. **Critical Thinking** If the length of the measurements of the net are correct to the nearest tenth of a centimeter, what is the maximum error in the surface area?

36. **Write About It** Explain how to use the net of a three-dimensional figure to find its surface area.
11.1-11B Review Homework

11-1 Solid Geometry
Classify each figure. Name the vertices, edges, and bases.
1.  \( \text{ABC} \)
2.  \( \text{H} \)
3.  \( \text{K} \)

Describe the three-dimensional figure that can be made from the given net.
4.  
5.  
6.  

Describe each cross section.
7.  
8.  
9.  

11A Representations of 3-D figures

10. Draw an isometric view of the figure.
   a) 
   b) 

11. Draw a triangular prism in one-point perspective.  
12. Draw a rectangular prism in two-point perspective.

11B  Surface Area of Prisms and Cylinders

Find the surface area of each figure. Round to the nearest tenth, if necessary.

13.  
14.  
15.  

16. The dimensions of a 12 mm by 8 mm by 24 mm right rectangular prism are multiplied by \( \frac{3}{4} \). Describe the effect on the surface area.
Objectives
Learn and apply the formula for the surface area of a pyramid.
Learn and apply the formula for the surface area of a cone.

Vocabulary
vertex of a pyramid
regular pyramid
slant height of a regular pyramid
altitude of a pyramid
vertex of a cone
right cone
oblique cone
slant height of a right cone
altitude of a cone

Why learn this?
A speaker uses part of the lateral surface of a cone to produce sound. Speaker cones are usually made of paper, plastic, or metal. (See Example 5.)

The vertex of a pyramid is the point opposite the base of the pyramid. The base of a regular pyramid is a regular polygon, and the lateral faces are congruent isosceles triangles. The slant height of a regular pyramid is the distance from the vertex to the midpoint of an edge of the base. The altitude of a pyramid is the perpendicular segment from the vertex to the plane of the base.

The lateral faces of a regular pyramid can be arranged to cover half of a rectangle with a height equal to the slant height of the pyramid. The width of the rectangle is equal to the base perimeter of the pyramid.

Lateral and Surface Area of a Regular Pyramid
The lateral area of a regular pyramid with perimeter $P$ and slant height $l$ is $L = \frac{1}{2}Pl$.
The surface area of a regular pyramid with lateral area $L$ and base area $B$ is $S = L + B$, or $S = \frac{1}{2}Pl + B$.

Example 1
Finding Lateral Area and Surface Area of Pyramids
Find the lateral area and surface area of each pyramid.

A: a regular square pyramid with base edge length 5 in. and slant height 9 in.
$L = \frac{1}{2}Pl$
$= \frac{1}{2}(20)(9) = 90 \text{ in}^2$
$S = \frac{1}{2}Pl + B$
$= 90 + 25 = 115 \text{ in}^2$

Lateral area of a regular pyramid
$P = 4(5) = 20 \text{ in.}$
Surface area of a regular pyramid
$B = 5^2 = 25 \text{ in}^2$
Find the lateral area and surface area of each regular pyramid. Round to the nearest tenth.

Step 1 Find the base perimeter and apothem.
The base perimeter is \(6(4) = 24\) m.
The apothem is \(2\sqrt{3}\) m, so the base area is \(\frac{1}{2}aP = \frac{1}{2}(2\sqrt{3})(24) = 24\sqrt{3}\) m².

Step 2 Find the lateral area.
\[L = \frac{1}{2}P\ell\]
\[= \frac{1}{2}(24)(7) = 84\] m²

Step 3 Find the surface area.
\[S = \frac{1}{2}P\ell + B\]
\[= 84 + 24\sqrt{3} \approx 125.6\] cm²

**Check It Out!**

1. Find the lateral area and surface area of a regular triangular pyramid with base edge length 6 ft and slant height 10 ft.

The **vertex of a cone** is the point opposite the base. The **axis of a cone** is the segment with endpoints at the vertex and the center of the base. The axis of a **right cone** is perpendicular to the base. The axis of an **oblique cone** is not perpendicular to the base.

The **slant height of a right cone** is the distance from the vertex of a right cone to a point on the edge of the base. The **altitude of a cone** is a perpendicular segment from the vertex of the cone to the plane of the base.

**Lateral and Surface Area of a Right Cone**

The lateral area of a right cone with radius \(r\) and slant height \(\ell\) is \(L = \pi\ell\).

The surface area of a right cone with lateral area \(L\) and base area \(B\) is \(S = L + B\), or \(S = \pi r\ell + \pi r^2\).
EXAMPLE 2

Finding Lateral Area and Surface Area of Right Cones

Find the lateral area and surface area of each cone. Give your answers in terms of $\pi$.

A. a right cone with radius 2 m and slant height 3 m

$L = \pi rl$

$= \pi (2)(3) = 6\pi$ m$^2$

$S = \pi rl + \pi r^2$

$= 6\pi + \pi (2)^2 = 10\pi$ m$^2$

Lateral area of a cone

Substitute 2 for $r$ and 3 for $l$.

Surface area of a cone

Substitute 2 for $r$ and 3 for $l$.

B.

Step 1 Use the Pythagorean Theorem to find $l$.

$l = \sqrt{5^2 + 12^2} = 13$ ft

Step 2 Find the lateral area and surface area.

$L = \pi rl$

$L = \pi (5)(13) = 65\pi$ ft$^2$

$L = \pi rl + \pi r^2$

$L = 65\pi + \pi (5)^2 = 90\pi$ ft$^2$

Lateral area of a right cone

Substitute 5 for $r$ and 13 for $l$.

Surface area of a right cone

Substitute 5 for $r$ and 13 for $l$.

CHECK IT OUT!

2. Find the lateral area and surface area of the right cone.

$L = 80\pi$ cm$^2$; $S \approx 144\pi$ cm$^2$

EXAMPLE 3

Exploring Effects of Changing Dimensions

The radius and slant height of the right cone are tripled. Describe the effect on the surface area.

original dimensions:

$S = \pi rl + \pi r^2$

$= \pi (3)(5) + \pi (3)^2$

$= 24\pi$ cm$^2$

radius and slant height tripled:

$S = \pi rl + \pi r^2$

$= \pi (9)(15) + \pi (9)^2$

$= 216\pi$ cm$^2$

Notice that $216\pi = 9(24\pi)$. If the length, width, and height are tripled, the surface area is multiplied by $3^2$, or 9.

CHECK IT OUT!

3. The base edge length and slant height of the regular square pyramid are both multiplied by $\frac{2}{3}$. Describe the effect on the surface area.

The surface area is multiplied by $\frac{4}{9}$.
EXAMPLE 4
Finding Surface Area of Composite Three-Dimensional Figures

Find the surface area of the composite figure.

The height of the cone is \( h = 90 - 45 = 45 \) cm.
By the Pythagorean Theorem,
\[ \ell = \sqrt{28^2 + 45^2} = 53 \text{ cm.} \]
The lateral area of the cone is
\[ L = \pi \ell \ell = \pi (28)(53) = 1484\pi \text{ cm}^2. \]
The lateral area of the cylinder is
\[ L = 2\pi rh = 2\pi (28)(45) = 2520\pi \text{ cm}^2. \]
The base area is
\[ B = \pi r^2 = \pi (28)^2 = 784\pi \text{ cm}^2. \]
The total surface area is
\[ S = (\text{cone lateral area}) + (\text{cylinder lateral area}) + (\text{base area}) \]
\[ = 2520\pi + 784\pi + 1484\pi = 4788\pi \text{ cm}^2. \]

CHECK IT OUT!
4. Find the surface area of the composite figure.
\[ S \approx 28.9 \text{ yd}^2 \]

EXAMPLE 5
Electronics Application

Tim is replacing the paper cone of an antique speaker. He measured the existing cone and created the pattern for the lateral surface from a large circle.

What is the diameter of the cone?

The radius of the large circle used to create the pattern is the slant height of the cone.
The area of the pattern is the lateral area of the cone. The area of the pattern is also \( \frac{3}{4} \) of the area of the large circle, so \( \pi \ell \ell = \frac{3}{4} \pi r^2 \).

\[ \pi \ell (10) = \frac{3}{4} \pi (10)^2 \]
Substitute 10 for \( \ell \), the slant height of the cone and the radius of the large circle. Solve for \( r \).

\[ r = 7.5 \text{ in.} \]
The diameter of the cone is \( 2(7.5) = 15 \) in.

CHECK IT OUT!
5. What if...? If the radius of the large circle were 12 in., what would be the radius of the cone? \( 9 \) in.

THINK AND DISCUSS

1. Explain why the lateral area of a regular pyramid is \( \frac{1}{2} \) the base perimeter times the slant height.

2. In a right cone, which is greater, the height or the slant height? Explain.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write the name of the part of the cone.
PRACTICE AND PROBLEM SOLVING

13. Find the lateral area and surface area of each regular pyramid.

14. Find the lateral area and surface area of each right cone. Give your answers in terms of $\pi$.

15. a regular hexagonal pyramid with base edge length 7 ft and slant height 15 ft

16. a cone with radius 8 m and height that is 1 m less than twice the radius

17. Describe the effect of each change on the surface area of the given figure.

18. The dimensions are divided by 3.

19. The dimensions are doubled.

20. Find the surface area of each composite figure.

21. 22.

23. It is a tradition in England to celebrate May 1st by hanging cone-shaped baskets of flowers on neighbors’ door handles. Addy is making a basket from a piece of paper that is a semicircle with diameter 12 in. What is the diameter of the basket?

Find the surface area of each figure.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Base Area</th>
<th>Slant Height</th>
<th>Surface Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>24. Regular square pyramid</td>
<td>36 cm$^2$</td>
<td>5 cm</td>
<td></td>
</tr>
<tr>
<td>25. Regular triangular pyramid</td>
<td>$\sqrt{3}$ m$^2$</td>
<td>$\sqrt{3}$ m</td>
<td></td>
</tr>
<tr>
<td>26. Right cone</td>
<td>$16\pi$ in$^2$</td>
<td>7 in.</td>
<td></td>
</tr>
<tr>
<td>27. Right cone</td>
<td>$\pi$ ft$^2$</td>
<td>2 ft</td>
<td></td>
</tr>
</tbody>
</table>
28. This problem will prepare you for the Multi-Step Test Prep on page 724. A juice container is a regular square pyramid with the dimensions shown.
   a. Find the surface area of the container to the nearest tenth.
   b. The manufacturer decides to make a container in the shape of a right cone that requires the same amount of material. The base diameter must be 9 cm. Find the slant height of the container to the nearest tenth.

29. Find the radius of a right cone with slant height 21 m and surface area $168\pi \text{ m}^2$.
30. Find the slant height of a regular square pyramid with base perimeter 32 ft and surface area $256 \text{ ft}^2$.
31. Find the base perimeter of a regular hexagonal pyramid with slant height 10 cm and lateral area $120 \text{ cm}^2$.
32. Find the surface area of a right cone with a slant height of 25 units that has its base centered at $(0, 0, 0)$ and its vertex at $(0, 0, 7)$.

Find the surface area of each composite figure.
33.  
   34.  

35. **Architecture** The Pyramid Arena in Memphis, Tennessee, is a square pyramid with base edge lengths of 200 yd and a height of 32 stories. Estimate the area of the glass on the sides of the pyramid. (*Hint: 1 story $\approx 10 \text{ ft})*

36. **Critical Thinking** Explain why the slant height of a regular square pyramid must be greater than half the base edge length.
37. **Write About It** Explain why slant height is not defined for an oblique cone.

38. Which expressions represent the surface area of the regular square pyramid?
   I. $\frac{t^2}{16} + \frac{ts}{2}$
   II. $\frac{t^2}{16} + \frac{tl}{2}$
   III. $\frac{t}{2}(8 + t)$
   A. I only  
   B. II only  
   C. I and II  
   D. II and III

39. A regular square pyramid has a slant height of 18 cm and a lateral area of 216 cm$^2$. What is the surface area?
   F. 252 cm$^2$    G. 234 cm$^2$    H. 225 cm$^2$    I. 240 cm$^2$

40. What is the lateral area of the cone?
   A. $360\pi \text{ cm}^2$  
   B. $369\pi \text{ cm}^2$  
   C. $450\pi \text{ cm}^2$  
   D. $1640\pi \text{ cm}^2$
11C Quiz Review

11C Surface Area of Pyramids and Cones
Find the surface area of each figure. Round to the nearest tenth, if necessary.
5. a regular pentagonal pyramid with base edge length 18 yd and slant height 20 yd
6. a right cone with diameter 30 in. and height 8 in.
7. the composite figure formed by two cones

11A-11C Test Review

Draw an isometric view of the figure.

8. 

9. 

10. Draw a pentagonal prism in one-point perspective.
Determine whether each drawing represents the given object. Assume there are no hidden cubes.

11. 

12. 

Find the lateral area and surface area of each right prism or cylinder. Round to the nearest tenth, if necessary.

19.

20. a cube with side length 5 ft

21. an equilateral triangular prism with height 7 m and base edge lengths 6 m

22. a regular pentagonal prism with height 8 cm and base edge length 4 cm

Find the lateral area and surface area of each right pyramid or cone.

23. a square pyramid with side length 15 ft and slant height 21 ft

24. a cone with radius 7 m and height 24 m

25. a cone with diameter 20 in. and slant height 15 in.

Find the surface area of each composite figure.

26. 

27. 

12 m
LESSON 10-2  Representations of Three-Dimensional Figures

An orthographic drawing of a three-dimensional object shows six different views of the object. The six views of the figure at right are shown below.

Top: [ diagram of top view ]  Bottom: [ diagram of bottom view ]  Front: [ diagram of front view ]

Back: [ diagram of back view ]  Left: [ diagram of left view ]  Right: [ diagram of right view ]

Draw all six orthographic views of each object. Assume there are no hidden cubes.

1. [ diagram of object ]

2. [ diagram of object ]
An **isometric drawing** is drawn on isometric dot paper and shows three sides of a figure from a corner view. A solid and an isometric drawing of the solid are shown.

In a **one-point perspective drawing**, nonvertical lines are drawn so that they meet at a **vanishing point**. You can make a one-point perspective drawing of a triangular prism.

**Step 1** Draw a horizontal line and a vanishing point on the line. Draw a triangle below the line.

**Step 2** From each vertex of the triangle, draw dashed segments to the vanishing point.

**Step 3** Draw a smaller triangle with vertices on the dashed segments.

**Step 4** Draw the edges of the prism. Use dashed lines for hidden edges. Erase segments that are not part of the prism.

Draw an isometric view of each object. Assume there are no hidden cubes.

3.

![Isometric View](image)

4.

![Isometric View](image)

Draw each object in one-point perspective.

5. a triangular prism with bases that are obtuse triangles

6. a rectangular prism
An isometric drawing is drawn on isometric dot paper and shows three sides of a figure from a corner view. A solid and an isometric drawing of the solid are shown.

In a one-point perspective drawing, nonvertical lines are drawn so that they meet at a vanishing point. You can make a one-point perspective drawing of a triangular prism.

Step 1: Draw a horizontal line and a vanishing point on the line. Draw a triangle below the line.

Step 2: From each vertex of the triangle, draw dashed segments to the vanishing point.

Step 3: Draw a smaller triangle with vertices on the dashed segments.

Step 4: Draw the edges of the prism. Use dashed lines for hidden edges. Erase segments that are not part of the prism.

Draw an isometric view of each object. Assume there are no hidden cubes.

3.

4.

Draw each object in one-point perspective.

5. a triangular prism with bases that are obtuse triangles

6. a rectangular prism
LESSON 10-2
Practice B
Representations of Three-Dimensional Figures

Draw all six orthographic views of each object. Assume there are no hidden cubes. In your answers, use a dashed line to show that the edges touch and a solid line to show that the edges do not touch.

1. [Diagram]

2. [Diagram]

3. Draw an isometric view of the object in Exercise 1.


5. Draw a block letter T in one-point perspective.

6. Draw a block letter T in two-point perspective. (Hint: Draw the vertical line segment that will be closest to the viewer first.)

Determine whether each drawing represents the object at right. Assume there are no hidden cubes.

7. Top [Diagram]
   Bottom [Diagram]
   Left [Diagram]
   Right [Diagram]
   Front [Diagram]
   Back [Diagram]

8. [Diagram]
**Practice A**

**Representations of Three-Dimensional Figures**

Draw all six orthographic views of each object (top, bottom, front, back, left, and right). Assume there are no hidden cubes. In your answers, use a dashed line to show that the edges do not touch.

1. 
   - Top: 
   - Bottom: 
   - Front: 
   - Back: 
   - Left:  
   - Right: 

2. 
   - Top: 
   - Bottom: 
   - Front: 
   - Back: 
   - Left:  
   - Right: 

In an isometric drawing, every corner of a cube is on a dot in the grid.

3. Draw an isometric view of the object in Exercise 1.

5. Follow the steps to complete the drawing of a prism in one-point perspective:
   - a. Draw a vertical line from each vertex of the triangle to the vanishing point (point V).
   - b. Use the dashed lines as guides to draw a triangle with sides parallel to the first triangle.
   - c. Connect corresponding vertices of the two triangles. Use dashed lines for all hidden edges.

Determine whether each drawing represents the object at right. Assume there are no hidden cubes.

6. Yes
7. No

**Practice B**

**Representations of Three-Dimensional Figures**

Draw all six orthographic views of each object. Assume there are no hidden cubes. In your answers, use a dashed line to show that the edges do not touch.

1. 
   - Top: 
   - Bottom: 
   - Front: 
   - Back: 
   - Left:  
   - Right: 

2. 
   - Top: 
   - Bottom: 
   - Front: 
   - Back: 
   - Left:  
   - Right: 

3. Draw an isometric view of the object in Exercise 1.

5. Draw a block letter T in one-point perspective. Possible answer:

6. Draw a block letter T in two-point perspective. (Hint: Draw the vertical line segment that will be closest to the viewer first.) Possible answer:

Determine whether each drawing represents the object at right. Assume there are no hidden cubes.

7. Yes
8. No

**Practice C**

**Representations of Three-Dimensional Figures**

Draw an isometric view of each object based on the orthographic views provided.

1. 
   - Top: 
   - Bottom: 
   - Front: 
   - Back: 
   - Left:  
   - Right: 

The object shown is made up of three pieces. Each piece is made of one or more adjoining cubes. Assume there are no hidden cubes.

3. Assume each piece has a different shape and at least one piece is not a rectangular prism. Draw 3-D representations of the pieces.

4. Combine the three pieces you drew in Exercise 3 to make a rectangular prism. Draw this prism and shade the pieces so they can be distinguished.

5. Now suppose that two of the three pieces have the same shape. Draw the two same-shaped pieces. Then draw six possibilities for the third piece.

6. Four of the six possibilities you drew in Exercise 5 can form a 2-by-2-by-2 cube when joined together with another identical piece. Draw such a cube and shade the two pieces so they can be distinguished.

Possible answer:

**Practice D**

**Representations of Three-Dimensional Figures**

An orthographic drawing of a three-dimensional object shows six different views of the object. The six views of the figure at right are shown below:

Top: 
Bottom: 
Front: 

Back: 
Left: 
Right: 

Draw all six orthographic views of each object. Assume there are no hidden cubes.

1. 
   - Top: 
   - Bottom: 
   - Front: 

Back: 
Left: 
Right: 

2. 
   - Top: 
   - Bottom: 
   - Front: 

Back: 
Left: 
Right: 

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Reteach

10-4 Surface Area of Prisms and Cylinders

The lateral area of a prism is the sum of the areas of all the lateral faces. A lateral face is not a base. The surface area is the total area of all faces.

<table>
<thead>
<tr>
<th>Lateral Area</th>
<th>The lateral area of a right prism with base perimeter (P) and height (h) is (L = Ph).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface Area</td>
<td>The surface area of a right prism with lateral area (L) and base area (B) is (S = L + 2B), or (S = Ph + 2B).</td>
</tr>
</tbody>
</table>

The lateral area of a right cylinder is the curved surface that connects the two bases. The surface area is the total area of the curved surface and the bases.

<table>
<thead>
<tr>
<th>Lateral Area</th>
<th>The lateral area of a right cylinder with radius (r) and height (h) is (L = 2\pi rh).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface Area</td>
<td>The surface area of a right cylinder with lateral area (L) and base area (B) is (S = L + 2B), or (S = 2\pi rh + 2\pi r^2).</td>
</tr>
</tbody>
</table>

Find the lateral area and surface area of each right prism.

1. [Diagram]

2. [Diagram]

Find the lateral area and surface area of each right cylinder. Give your answers in terms of \(\pi\).

3. [Diagram]

4. [Diagram]
LESSON 10-4 Reteach
Surface Area of Prisms and Cylinders continued

You can find the surface area of a composite three-dimensional figure like the one shown at right.

\[
\text{surface area of small prism} + \text{surface area of large prism} - \text{hidden surfaces}
\]

The dimensions are multiplied by 3. Describe the effect on the surface area.

original surface area: 
\[
S = Ph + 2B \\
= 20(3) + 2(16) \\
= 92 \text{ mm}^2
\]

new surface area, dimensions multiplied by 3: 
\[
S = Ph + 2B \\
= 60(9) + 2(144) \\
= 828 \text{ mm}^2
\]

Notice that 92 \cdot 9 = 828. If the dimensions are multiplied by 3, the surface area is multiplied by 3^2, or 9.

Find the surface area of each composite figure. Be sure to subtract the hidden surfaces of each part of the composite solid. Round to the nearest tenth.

5. 

6. 

Describe the effect of each change on the surface area of the given figure.

7. The length, width, and height are multiplied by 2.

8. The height and radius are multiplied by \(\frac{1}{2}\).
Reteach

**Surface Area of Prisms and Cylinders** continued

You can find the surface area of a composite three-dimensional figure like the one shown at right.

\[
\text{surface area of small prism} + \text{surface area of large prism} - \text{hidden surfaces}
\]

The dimensions are multiplied by 3. Describe the effect on the surface area.

\[
\text{original surface area: } S = Ph + 2B \\
= 20(3) + 2(16) \quad P = 20, \ h = 3, \ B = 16 \\
= 92 \text{ mm}^2 \quad \text{Simplify.}
\]

\[
\text{new surface area, dimensions multiplied by 3: } S = Ph + 2B \\
= 60(9) + 2(144) \quad P = 60, \ h = 9, \ B = 144 \\
= 828 \text{ mm}^2 \quad \text{Simplify.}
\]

Notice that \(92 \cdot 9 = 828\). If the dimensions are multiplied by 3, the surface area is multiplied by \(3^2\), or 9.

Find the surface area of each composite figure. Be sure to subtract the hidden surfaces of each part of the composite solid. Round to the nearest tenth.

5. \[
\begin{array}{c}
\text{original surface area: } S = Ph + 2B \\
= 20(3) + 2(16) \quad P = 20, \ h = 3, \ B = 16 \\
= 92 \text{ mm}^2 \quad \text{Simplify.}
\end{array}
\]

6. \[
\begin{array}{c}
\text{new surface area, dimensions multiplied by 3: } S = Ph + 2B \\
= 60(9) + 2(144) \quad P = 60, \ h = 9, \ B = 144 \\
= 828 \text{ mm}^2 \quad \text{Simplify.}
\end{array}
\]

Describe the effect of each change on the surface area of the given figure.

7. The length, width, and height are multiplied by 2.

8. The height and radius are multiplied by \(\frac{1}{2}\).
Lesson 10-4
Surface Area of Prisms and Cylinders

Find the lateral area and surface area of each right prism. Round to the nearest tenth if necessary.

1. the rectangular prism

2. the regular pentagonal prism

3. a cube with edge length 20 inches

Find the lateral area and surface area of each right cylinder. Give your answers in terms of \( \pi \).

4. 

5. a cylinder with base area 169\( \pi \) ft\(^2\) and a height twice the radius

6. a cylinder with base circumference 8\( \pi \) m and a height one-fourth the radius

Find the surface area of each composite figure. Round to the nearest tenth.

7. 

8. 

Describe the effect of each change on the surface area of the given figure.

9. The dimensions are multiplied by 12.

10. The dimensions are divided by 4.

Toby has eight cubes with edge length 1 inch. He can stack the cubes into three different rectangular prisms: 2-by-2-by-2, 8-by-1-by-1, and 2-by-4-by-1. Each prism has a volume of 8 cubic inches.

11. Tell which prism has the smallest surface-area-to-volume ratio.

12. Tell which prism has the greatest surface-area-to-volume ratio.
**Practice A**

**Surface Area of Prisms and Cylinders**

Write each formula.
1. Lateral area of a right prism with base perimeter \( P \) and height \( h \)
   \[ L = Ph \]

2. Lateral area of a right cylinder with radius \( r \) and height \( h \)
   \[ L = 2\pi rh \]

3. Surface area of a right prism with lateral area \( L \) and base area \( B \)
   \[ S = L + 2B \]

4. Surface area of a cube with edge length \( s \)
   \[ S = 6s^2 \]

5. Surface area of a right cylinder with radius \( r \) and height \( h \)
   \[ S = 2\pi rh + 2\pi r^2 \]

Find the lateral area and surface area of each right prism.

- **Problem 1:**
  - Given data: \( L = 120 \text{ cm}^2 \), \( S = 168 \text{ cm}^2 \)
  - Find the lateral area and surface area.

- **Problem 2:**
  - Given data: \( L = 150 \text{ in}^2 \), \( S = 240 \text{ in}^2 \)
  - Find the lateral area and surface area.

- **Problem 3:**
  - Given data: \( L = 18 \text{ ft}^2 \), \( S = 80 \text{ ft}^2 \)
  - Find the lateral area and surface area.

- **Problem 4:**
  - Given data: \( L = 8 \text{ in}^2 \), \( S = 16 \text{ in}^2 \)
  - Find the lateral area and surface area.

- **Problem 5:**
  - Given data: \( L = 60 \text{ cm}^3 \), \( S = 110 \text{ cm}^3 \)
  - Find the lateral area and surface area.

- **Problem 6:**
  - Given data: \( L = 160 \text{ mm}^2 \), \( S = 83.8 \text{ mm}^2 \)
  - Find the lateral area and surface area.

**Practice B**

**Surface Area of Prisms and Cylinders**

Find the lateral area and the surface area of each right prism. Round to the nearest tenth if necessary.

- **Problem 1:**
  - Given data: \( L = 176 \text{ m}^2 \), \( S = 416 \text{ m}^2 \)
  - Find the lateral area and surface area.

- **Problem 2:**
  - Given data: \( L = 78 \text{ in}^2 \), \( S = 83.8 \text{ in}^2 \)
  - Find the lateral area and surface area.

- **Problem 3:**
  - Given data: \( L = 1600 \text{ in}^2 \), \( S = 2400 \text{ in}^2 \)
  - Find the lateral area and surface area.

**Practice C**

**Surface Area of Prisms and Cylinders**

A builder drills a hole through a cube of concrete, as shown in the figure. The cube will be an outlet for a water tap on the side of a house. Complete Exercises 11–14 to find the surface area of the figure. Round to the nearest tenth if necessary.

- **Problem 1:**
  - Given data: \( L = 8 \text{ in}^2 \), \( S = 16 \text{ in}^2 \)
  - Find the surface area.

- **Problem 2:**
  - Given data: \( L = 60 \text{ cm}^3 \), \( S = 78 \text{ cm}^3 \)
  - Find the surface area.

- **Problem 3:**
  - Given data: \( L = 384 \text{ in}^2 \)
  - Find the surface area.

- **Problem 4:**
  - Given data: \( L = 59.2 \text{ in}^2 \)
  - Find the surface area.

**Retake**

**Surface Area of Prisms and Cylinders**

The lateral area of a prism is the sum of the areas of all the lateral faces. A lateral face is not a base. The surface area is the total area of all faces.

**Lateral and Surface Area of a Right Prism**

**Lateral Area**

\[ L = Ph \]

**Surface Area**

\[ S = L + 2B \]

The lateral area of a right cylinder is the curved surface that connects the two bases. The surface area is the total area of the curved surface and the bases.

**Lateral and Surface Area of a Right Cylinder**

**Lateral Area**

\[ L = 2\pi rh \]

**Surface Area**

\[ S = L + 2\pi r^2 \]

Find the lateral area and surface area of each right prism.

- **Problem 1:**
  - Given data: \( L = 78 \text{ ft}^2 \), \( S = 110 \text{ ft}^2 \)
  - Find the lateral area and surface area.

- **Problem 2:**
  - Given data: \( L = 8 \text{ in}^2 \), \( S = 416 \text{ in}^2 \)
  - Find the lateral area and surface area.

- **Problem 3:**
  - Given data: \( L = 60 \text{ in}^2 \), \( S = 110 \text{ in}^2 \)
  - Find the lateral area and surface area.
**LESSON**

**10-5**

**Reteach**

**Surface Area of Pyramids and Cones**

**Lateral and Surface Area of a Regular Pyramid**

<table>
<thead>
<tr>
<th>Lateral Area</th>
<th>The lateral area of a regular pyramid with perimeter ( P ) and slant height ( \ell ) is ( L = \frac{1}{2}P\ell ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface Area</td>
<td>The surface area of a regular pyramid with lateral area ( L ) and base area ( B ) is ( S = L + B ), or ( S = \frac{1}{2}P\ell + B ).</td>
</tr>
</tbody>
</table>

**Lateral and Surface Area of a Right Cone**

<table>
<thead>
<tr>
<th>Lateral Area</th>
<th>The lateral area of a right cone with radius ( r ) and slant height ( \ell ) is ( L = \pi r\ell ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface Area</td>
<td>The surface area of a right cone with lateral area ( L ) and base area ( B ) is ( S = L + B ), or ( S = \pi r\ell + \pi r^2 ).</td>
</tr>
</tbody>
</table>

Find the lateral area and surface area of each regular pyramid. Round to the nearest tenth.

1.  
   ![Pyramid 1](image1.png)

2.  
   ![Pyramid 2](image2.png)

Find the lateral area and surface area of each right cone. Give your answers in terms of \( \pi \).

3.  
   ![Cone 1](image3.png)

4.  
   ![Cone 2](image4.png)
LESSON 10-5 Reteach
Surface Area of Pyramids and Cones continued

The radius and slant height of the cone at right are doubled. Describe the effect on the surface area.

original surface area:
\[ S = \pi rl + \pi r^2 \]
\[ = \pi (3)(7) + \pi (3)^2 \text{ \hspace{1cm} } r = 3, \ell = 7 \]
\[ = 30\pi \text{ cm}^2 \text{ \hspace{1cm} } \text{Simplify.} \]

new surface area, dimensions doubled:
\[ S = \pi rl + \pi r^2 \]
\[ = \pi (6)(14) + \pi (6)^2 \text{ \hspace{1cm} } r = 6, \ell = 14 \]
\[ = 120\pi \text{ cm}^2 \text{ \hspace{1cm} } \text{Simplify.} \]

If the dimensions are doubled, then the surface area is multiplied by \(2^2\), or 4.

Describe the effect of each change on the surface area of the given figure.

5. The dimensions are tripled.

6. The dimensions are multiplied by \(\frac{1}{2}\).

Find the surface area of each composite figure.

7. \textit{Hint:} Do not include the base area of the pyramid or the upper surface area of the rectangular prism.

8. \textit{Hint:} Add the lateral areas of the cones.
Reteach

LESSON 10-5 Surface Area of Pyramids and Cones continued

The radius and slant height of the cone at right are doubled. Describe the effect on the surface area.

Original surface area:

\[ S = \pi r l + \pi r^2 \]
\[ = \pi (3)(7) + \pi (3)^2 \quad r = 3, \ l = 7 \]
\[ = 30\pi \text{ cm}^2 \quad \text{Simplify} \]

New surface area, dimensions doubled:

\[ S = \pi r l + \pi r^2 \]
\[ = \pi (6)(14) + \pi (6)^2 \quad r = 6, \ l = 14 \]
\[ = 120\pi \text{ cm}^2 \quad \text{Simplify} \]

If the dimensions are doubled, then the surface area is multiplied by \(2^2\), or 4.

Describe the effect of each change on the surface area of the given figure.

5. The dimensions are tripled.

6. The dimensions are multiplied by \(\frac{1}{2}\).

Find the surface area of each composite figure.

7. **Hint:** Do not include the base area of the pyramid or the upper surface area of the rectangular prism.

8. **Hint:** Add the lateral areas of the cones.
LESSON 10-5
Surface Area of Pyramids and Cones

Find the lateral area and surface area of each regular right solid. Round to the nearest tenth if necessary.

1. \[ \text{20 yd} \]
   \[ \text{96 yd} \]

2. \[ \text{18 m} \]
   \[ \text{9 m} \]

3. a regular hexagonal pyramid with base edge length 12 mi and slant height 15 mi

Find the lateral area and surface area of each right cone. Give your answers in terms of \( \pi \).

4.

5. a right cone with base circumference \( 14 \pi \) ft and slant height 3 times the radius

6. a right cone with diameter 240 cm and altitude 35 cm

Describe the effect of each change on the surface area of the given figure.

7. The dimensions are multiplied by \( \frac{1}{5} \).

8. The dimensions are multiplied by \( \frac{3}{2} \).

Find the surface area of each composite figure. Round to the nearest tenth if necessary.

9.

10.

11. The water cooler at Mohammed's office has small conical paper cups for drinking. He uncurls one of the cups and measures the paper. Based on the diagram of the uncurled cup, find the diameter of the cone.
**Practice A**

**165 Surface Area of Pyramids and Cones**

Write each formula.

1. Lateral area of a right pyramid with base perimeter $P$ and slant height $h$: $L = \frac{1}{2} Pl$

2. Lateral area of a right cone with radius $r$ and slant height $h$: $S = \pi rh$

3. Surface area of a regular pyramid with lateral area $L$ and base area $B$: $S = L + B$

4. Surface area of a right cone with lateral area $L$ and base area $B$: $S = L + B$

Find the lateral area and surface area of each regular pyramid. Round to the nearest tenth if necessary.

5. The regular square pyramid
   - Lateral area: $L = 78 \text{ cm}^2$
   - Surface area: $S = 85 \text{ cm}^2$

Find the lateral area and surface area of each right cone. Give your answers in terms of $n$.

6. A right cone with radius $r$ and slant height $s$:
   - Lateral area: $L = \pi rs$
   - Surface area: $S = \pi r^2 + \pi rs$

Complete Exercises 5-11 to describe the effect on the surface area of dividing the dimensions of a cone by 2. Give your answers in terms of $n$.

7. Find the surface area of a right cone with radius 2 $n$ and slant height 6 $n$.
   - Lateral area: $L = 24 \pi n^2$
   - Surface area: $S = 36 \pi n^2$

8. Find the surface area of a right cone with radius 1 $n$ and slant height 3 $n$.
   - Lateral area: $L = 3 \pi n^2$
   - Surface area: $S = 6 \pi n^2$

9. Describe the effect on the surface area of doubling the dimensions of a right cone by 2.
   - The surface area is divided by 4.

10. Find the surface area of the composite figure in terms of $n$.
    - $124 \pi n^2$

**Practice B**

**165 Surface Area of Pyramids and Cones**

Find the lateral area and surface area of each regular right solid. Round to the nearest tenth if necessary.

1. A regular pentagonal pyramid with base area $12 \text{ in}^2$ and slant height $10 \text{ in}$:
   - Lateral area: $5 \cdot 12 \pi = 60 \pi \text{ in}^2$
   - Surface area: $S = 60 \pi + 12 \pi = 72 \pi \text{ in}^2$

2. A regular hexagonal pyramid with base area $18 \text{ in}^2$ and slant height $9 \text{ in}$:
   - Lateral area: $6 \cdot 18 \pi = 108 \pi \text{ in}^2$
   - Surface area: $S = 108 \pi + 18 \pi = 126 \pi \text{ in}^2$

3. A regular octagonal pyramid with base area $24 \text{ in}^2$ and slant height $12 \text{ in}$:
   - Lateral area: $8 \cdot 24 \pi = 192 \pi \text{ in}^2$
   - Surface area: $S = 192 \pi + 24 \pi = 216 \pi \text{ in}^2$

4. A regular decagonal pyramid with base area $30 \text{ in}^2$ and slant height $15 \text{ in}$:
   - Lateral area: $10 \cdot 30 \pi = 300 \pi \text{ in}^2$
   - Surface area: $S = 300 \pi + 30 \pi = 330 \pi \text{ in}^2$

**Reteach**

**165 Surface Area of Pyramids and Cones**

Use the figure for Exercises 1-3. The figure shown can be cut into an open-top box.

1. Find the radius of the top of the cone.

2. Find the radius of a circle that has the same area as the lateral area of the cone.

3. Name the mathematical relationship between the answer in Exercise 2 and the radius and slant height of the cone.

Find the radius of a circle that has the same area as the lateral area of each cone described in Exercises 4-6. Give exact answers.

4. L = $16 \pi$, $r = ?$, $h = 8$, $l = 4$
   - $r = \sqrt{2}$

5. A cone with a base radius can be formed by an open partial circle. Develop a formula for the surface area of the cone.
   - $S = \pi r^2 + \pi rl$

6. The figures below can be cut into open boxes. Find the lateral area of the cone and the radius of both bases in each frustum. Round to the nearest tenth.

   - $L = 16.8 \pi$, $r_1 = 0.3 \text{ in}$, $r_2 = 1.8$
   - $L = 17.3 \pi$, $r_1 = 2.4\pi$, $r_2 = 3.1 \text{ in}$

7. Terry is driving through the desert when she notices the engine is low on oil. She has a few quarts of motor oil in the trunk of the car, but she does not have a funnel. Fortunately, Terry finds a piece of 8 1/2 by 11-inch notebook paper in the backseat. She needs to cut out a funnel. Make a funnel with a 1-inch diameter hole at the bottom and the longest slant height possible. Find the diameter of the top of the funnel. Draw the pattern Terry will cut out before cutting up the funnel.
   - $\frac{5}{2} \text{ in}$
**Practice A**

**Representations of Three-Dimensional Figures**

Draw all six orthographic views of each object (top, bottom, front, back, left, and right). Assume there are no hidden cubes. In your answers, use a dashed line to show that the edges touch and a solid line to show that the edges do not touch.

1. Top Bottom Front Back Left Right

2. Top Bottom Left

3. Draw an isometric view of the object in Exercise 1.


5. Follow the steps to complete the drawing of a triangular prism in one-point perspective.
   a. Draw a dashed line from each vertex of the triangle to the vanishing point (point V).
   b. Use the dashed lines as guides to draw a triangle with sides parallel to the flat triangle.
   c. Connect corresponding vertices of the two triangles. Use dashed lines for all hidden edges.

Determine whether each drawing represents the object at all. Assume there are no hidden cubes.

6. (Yes)

7. (No)

---

**Practice B**

**Representations of Three-Dimensional Figures**

Draw all six orthographic views of each object. Assume there are no hidden cubes. In your answers, use a dashed line to show that the edges touch and a solid line to show that the edges do not touch.

1. Top Bottom Front Back Left Right

2. Top Bottom Front

3. Draw an isometric view of the object in Exercise 1.


5. Draw a block letter T in one-point perspective. Possible answer:

6. Draw a block letter T in two-point perspective. Possible answer:

Determine whether each drawing represents the object at all. Assume there are no hidden cubes.

7. (Yes)

8. (No)

---

**Practice C**

**Representations of Three-Dimensional Figures**

Draw an isometric view of each object based on the orthographic views provided.

1. Front Top

2. Top Right Left

The object shown is made up of three pieces. Each piece is made of one or more adjacent cubes. Assume there are no hidden cubes.

3. Assume each piece has a different shape and at least one piece is not a rectangular prism. Draw 3-D representations of the pieces.

4. Combine the three pieces you drew in Exercise 3 to make a rectangular prism. Draw the prism and shade the pieces so they can be distinguished.

5. Draw a sequence that two of the three pieces have the same shape. Draw the two same-shaped pieces. Then draw the third piece so it can be distinguished.

6. Four of the six possibilities you drew in Exercise 5 can form a 2-by-2-by-2 cube when joined together with another identical piece. Draw each cube and shade the two pieces so they can be distinguished.

**Practice D**

**Representations of Three-Dimensional Figures**

An orthographic drawing of a three-dimensional object shows six different views of the object. The six views of the figure at right are shown below.

Top: 

Bottom: 

Front: 

Back: 

Left: 

Right: 

Draw all six orthographic views of each object. Assume there are no hidden cubes.

1. Top: 

   Bottom: 

   Front: 

   Back: 

   Left: 

   Right: 

2. Top: 

   Bottom: 

   Front: 

   Back: 

   Left: 

   Right: 

---

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Reteach

Representations of Three-Dimensional Figures continued

An isometric drawing is drawn on isometric dot paper and shows three sides of a figure from a corner view. A solid and an isometric drawing of the solid are shown.

In a one-point perspective drawing, nonvertical lines are drawn so that they meet at a vanishing point. You can make a one-point perspective drawing of a triangular prism.

Step 1: Draw a horizontal line and a vanishing point on the line. Draw a triangle below the line.

Step 2: From each vertex of the triangle, draw dashed segments to the vanishing point.

Step 3: Draw a smaller triangle with vertices on the dashed segments.

Step 4: Draw the edges of the prism. Use dashed lines for hidden edges. These segments that are not part of the prism.

Draw an isometric view of each object. Assume there are no hidden cubes.

3. 

4. 

5. A triangular prism with bases that are oblique triangles

6. A rectangular prism

Possible answer:

Possible answer:

Challenge

Investigating Antiprisms

On this page, you will work with a type of polyhedron called an antiprism.

1. Trace the pattern below onto heavy paper or cardboard. Cut out the pattern and color along the dashed lines. Then use glue or tape to assemble 1. The figure is a model of a right square antiprism. Check students work.

2. How is the right square antiprism like a right square prism?

Name as many similarities as you can.

Answers may vary. Each has two congruent, parallel square bases. In each, the segment whose endpoints are the centers of the bases is perpendicular to both bases. In each, all the diagonals are congruent to each other.

3. How is the right square antiprism different from a right square prism?

Name as many differences as you can.

Answers may vary.

4. On a separate sheet of paper, make a pattern for a right antiprism with three faces that are regular pentagons. Cut out and assemble the pattern. The figure is a right regular pentagonal antiprism. Patterns may vary slightly.

Problem Solving

Representations of Three-Dimensional Figures

1. Describe the top, front, and side views of the figure.

2. Draw a one-point perspective drawing of the figure.

3. Which is the best statement about the figure?

4. Which of the following views correctly represents the top view of the three-dimensional figure?

Choose the best answer.

5. Which of the following best represents the top view of the three-dimensional figure?

Assume there are no hidden cubes.

6. Which of the following best represents the side view of the building shown?

Complete the following.

1. What are the orthographic views of a three-dimensional object called?

They show the three-dimensional object from six different perspectives.

2. Draw the six orthographic views of the object shown at right and label each view. Assume there are no hidden cubes.

Reading Strategies

Use a Concept Map

Orthographic views show three-dimensional objects from six different perspectives. Use the concept map to help you visualize orthographic views.

Top

Observe the figure from the top.

Front

Observe the figure from the front.

Right

Observe the figure from the right.

Bottom

Observe the figure from the bottom.

Left

Observe the figure from the left.

Back

Observe the figure from the back.

Assume there are no hidden cubes.
### Practice A

**Surface Area of Prisms and Cylinders**

**Write each formula.**

1. lateral area of a right prism with base perimeter $p$ and height $h$
   \[ L = Ph \]
2. lateral area of a right cylinder with radius $r$ and height $h$
   \[ L = 2\pi rh \]
3. surface area of a right prism with lateral area $L$ and base area $B$
   \[ S = L + 2B \]
4. surface area of a cube with edge length $s$
   \[ S = 6s^2 \]
5. surface area of a right cylinder with radius $r$ and height $h$
   \[ S = 2\pi rh + 2\pi r^2 \]

**Find the lateral area and surface area of each right prism.**

- a rectangular prism: $L = 120 \text{ cm}^2$, $S = 168 \text{ cm}^2$
- a cube with edge length 4 ft: $L = 16 \text{ ft}^2$, $S = 24 \text{ ft}^2$

**Find the lateral area and surface area of each right cylinder.**

- a cylinder with radius 3 mm and a height of 10 mm: $L = 60\pi \text{ mm}^2$, $S = 78\pi \text{ mm}^2$

A builder drills a hole through a cube of concrete, as shown in the figure. This cube will be on an outlet for a water tap on the side of a house. Complete Exercises 11-14 to find the surface area of the figure. Round to the nearest tenth if necessary.

11. Find the surface area of the cube.
12. Find the lateral area of the cylinder.
13. Find the base area of the cylinder.
14. The surface area of the figure is the surface area of the prism plus the lateral area of the cylinder minus twice the base area of the cylinder. Find the surface area of the figure.

**Practice B**

**Surface Area of Prisms and Cylinders**

Find the lateral area and surface area of each right prism. Round to the nearest tenth if necessary.

- a rectangular prism: $L = 176 \text{ m}^2$, $S = 416 \text{ m}^2$
- a cube with edge length 20 inches $L = 1600 \text{ in}^2$, $S = 2400 \text{ in}^2$

**Find the lateral area and surface area of each right cylinder.**

- a cylinder with base area $10\pi \text{ in}^2$ and a height of 6 inches
- a cylinder with base circumference $6\pi$ m and a height equal to the radius

**Find the surface area of each composite figure. Round to the nearest tenth.**

- $L = 52 \text{ in}^2$, $S = 2153.1 \text{ in}^2$
- $L = 46 \text{ cm}^2$, $S = 1156.1 \text{ cm}^2$
- $L = 45.7 \text{ mm}^2$, $S = 92.3 \text{ mm}^2$

**Practice C**

**Surface Area of Prisms and Cylinders**

A heat sink is a check of metal that stores unused heat from delicate electronic components and releases the heat into the air. The figure shows a typical heat sink for a desktop computer processor chip. Each side is 2 cm wide and 4 cm long from the heat sink.

1. Find the surface area of the heat sink in square millimeters.

- $S = 31,840 \text{ mm}^2$

2. Explain why the heat sink has fins. Possible answer: The fins greatly increase the surface area of the heat sink. The large surface area allows the heat to be radiated into the air more rapidly.

Find the surface area of each figure. Round to the nearest tenth if necessary.

- the rectangular prism with no top: $L = 48 \text{ yd}^2$
- the right triangular prism: $L = 2153.1 \text{ m}^2$
- the cylinder with radius 3 yards and each edge measuring 3 yards: $L = 1156.1 \text{ in}^2$
- the oblique cylinder: $L = 1156.1 \text{ in}^2$

- $L = 48 \text{ yd}^2$, $S = 92.3 \text{ mm}^2$
- $L = 45.7 \text{ mm}^2$, $S = 92.3 \text{ mm}^2$

**Reconnect**

**Surface Area of Prisms and Cylinders**

The lateral area of a cylinder is the sum of the areas of all the lateral faces. A lateral face is not a base. The surface area is the total area of all faces.

**Lateral and Surface Area of a Right Prism**

<table>
<thead>
<tr>
<th>Lateral Area</th>
<th>Surface Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L = Ph$</td>
<td>$S = Ph + 2B$</td>
</tr>
</tbody>
</table>

**Lateral and Surface Area of a Right Cylinder**

<table>
<thead>
<tr>
<th>Lateral Area</th>
<th>Surface Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L = \pi rh$</td>
<td>$S = \pi r^2 + 2\pi rh$</td>
</tr>
</tbody>
</table>

Find the lateral area and surface area of each right prism.

- a rectangular prism: $L = 176 \text{ m}^2$, $S = 416 \text{ m}^2$
- a cube with edge length 20 inches $L = 1600 \text{ in}^2$, $S = 2400 \text{ in}^2$

Find the lateral area and surface area of each right cylinder. Give your answers in terms of \( \pi \).

- a cylinder with base area $10\pi \text{ in}^2$ and a height of 6 inches
- a cylinder with base circumference $6\pi$ m and a height equal to the radius

Find the surface area of each composite figure. Round to the nearest tenth.

- $L = 129.7 \text{ km}^2$, $S = 113.7 \text{ km}^2$
- $L = 129.7 \text{ km}^2$, $S = 113.7 \text{ km}^2$

Describe the effect of each change on the surface area of the given figure.

- The dimensions are multiplied by 12.
- The dimensions are divided by 4.

The surface area is multiplied by 144. The surface area is divided by 16.

The cylinder has a height of 1 inch. The top is a square with edge length 1 inch. The height is 1 inch. Each prism has a volume of 8 cubic inches.

- Find the distance that is the smallest surface area to volume ratio.
- Find the distance that is the greatest surface area to volume ratio.
**Challenges**

1. **Surface Area and Volume of Semiregular Polyhedra**

A semiregular polyhedron is a polyhedron whose faces are bounded by two or more types of regular polygons in such a way that the arrangement of polygons at each vertex of the polyhedron is identical.

**Figure 1:** A semiregular polyhedron called a cuboctahedron has faces bounded by equilateral triangles and squares. You can think of it as the figure obtained if you cut off eight congruent pieces from a cube in the manner shown.

1. How many square faces does a cuboctahedron have?
2. How many triangular faces does a cuboctahedron have?
3. What type of figure is each place that is cut off from the original cube?

**Suppose that the length of each edge of a cuboctahedron is 10 inches.**

**4.** What is the area of each square face?

**5.** What is the area of each triangular face?

**6.** What is the total surface area?

**7.** What is the volume of this cube?

**8.** What is the volume of each piece that was cut off from the cube?

**9.** What is the volume of the cuboctahedron?

**Distinguish your results from Exercise 3 and 4 to write formulas for the surface area and volume of a cuboctahedron with edge length**

\[ S = 6a^2 + 2a^2 \sqrt{3} \]

\[ V = \frac{3}{2} a^3 \sqrt{2} \]

**6.** When right congruent pieces are cut from a cube in the manner shown, the result is a semiregular polyhedron called a truncated cube. Write formulas for the surface area and volume of a truncated cube with edge length **m**.

\[ S = 12m^2 + 12m^2 \sqrt{2} + 2m^2 \sqrt{3} \]

\[ V = 7m^3 + 14m^3 \sqrt{2} \]

**Reading Strategies**

**1.** **Use a Concept Map**

Use the concept maps below to help you understand and use lateral and surface area formulas.

**Formulas**

- \( L = Ph \)  
- \( S = L + 2B = Ph + 2B \)  
- \( \text{Perimeter of base, } h = \text{height, and } B = \text{area of base} \)

**Diagram**

**Lateral Area** and **Surface Area of a Right Prism**

\[ L = (D + d)h = 227 \text{ cm}^2 \]

\[ B = \frac{45 \times 76}{2} = \text{78 cm}^2 \]

\[ S = 227 + 140 = 367 \text{ cm}^2 \]

**Lateral Area and Surface Area of a Right Cylinder**

\[ L = 2 \pi rh = 96 \pi \text{ cm}^2 \]

\[ B = \pi r^2 = \text{16 \pi cm}^2 \]

\[ S = 96 \pi + 32 \pi = \text{128 \pi cm}^2 \]

Find the lateral area and surface area of each figure. Round to the nearest tenth if necessary:

1. **Flat object**

\[ \ell = \text{60 cm} \]

\[ S = \text{566.3 cm}^2 \]

\[ S = \text{393.2 cm}^2 \]

2. **Cone**

\[ \ell = \text{36 cm} \]

\[ S = \text{785.4 m}^2 \]

3. **Pyramid**

\[ \ell = \text{60 m} \]

\[ S = \text{942.5 m}^2 \]
Practice A

Surface Area of Pyramids and Cones

Write each formula.
1. Lateral area of a regular pyramid with base perimeter P and slant height s.
   \[ \text{L} = \frac{1}{2} \text{PH} \]
2. Lateral area of a right cone with radius r and slant height s.
   \[ \text{L} = \pi r s \]
3. Surface area of a regular pyramid with lateral area L and base area B.
   \[ \text{S} = L + B \]
4. Surface area of a right cone with lateral area L and base area B.
   \[ \text{S} = L + B \]

Find the lateral area and surface area of each regular pyramid. Round to the nearest tenth if necessary.

5. \[ \text{L} = 70 \text{ cm}^2; \text{S} = 85 \text{ cm}^2 \]
6. \[ \text{L} = 68 \text{ in}^2; \text{S} = 87.7 \text{ in}^2 \]

Find the lateral area and surface area of each right cone. Give your answers in terms of \( \pi \).

7. \[ \text{L} = 20 \pi r^2; \text{S} = 35 \pi \text{ in}^2 \]
8. \[ \text{L} = 36 \pi \text{ m}^2; \text{S} = 45 \pi \text{ m}^2 \]

Complete Exercises 9-11 to describe the effect on the surface area of doubling the dimensions of a cone by 2. Give your answers in terms of \( \pi \).
9. Find the surface area of a right cone with radius 0.3 yards and slant height 3 yards.
   \[ 18 \pi \text{ yd}^2 \]
10. Find the surface area of a right cone with radius 1 yard and slant height 3 yards.
    \[ 4 \pi \text{ yd}^2 \]
11. Describe the effect on the surface area of doubling the dimensions of a right cone by 2.
    The surface area is divided by 4.

12. Find the surface area of the composite figure in terms of \( \pi \).
    \[ 124 \pi \text{ mm}^2 \]

Practice B

Surface Area of Pyramids and Cones

Find the lateral area and surface area of each regular right solid. Round to the nearest tenth if necessary.

1. \[ \text{L = 36 in}^2; \text{S = 45 in}^2 \]
2. \[ \text{L = 5994 yd}^2; \text{S = 19,200 yd}^2 \]
3. \[ \text{L = 405 m}^2; \text{S = 544.4 m}^2 \]
4. A regular hexagonal pyramid with base edge length 12 m and slant height 16 m.
   \[ \text{L = 540 m}^2; \text{S = 814 m}^2 \]
5. Find the lateral area and surface area of each right cone. Give your answers in terms of \( \pi \).
   \[ \text{L = 200 \pi \text{ km}}^2; \text{S = 265 \pi \text{ km}}^2 \]
6. A right cone with base circumference 14 \text{ ft} and slant height 3 times the radius.
   \[ \text{L = 147 \pi \text{ ft}}^2; \text{S = 105 \pi \text{ ft}}^2 \]
7. A right cone with diameter 24 cm and altitude 30 cm.
   \[ \text{L = 15,008 \pi \text{ cm}^2}; \text{S = 29,408 \pi \text{ cm}^2} \]

Describe the effect of each change on the surface area of the given figure.

8. The dimensions are multiplied by \( \frac{1}{2} \).
   The surface area is multiplied by \( \frac{1}{4} \).
9. The dimensions are multiplied by \( \frac{3}{2} \).
   The surface area is multiplied by \( \frac{9}{4} \).

10. Find the surface area of each composite figure. Round to the nearest tenth if necessary.
    \[ \text{L = 80 \text{ m}^2}; \text{S = 78.6 \text{ m}^2} \]

11. The water cooler at Uncommon's office is a small conical paper cup for drinking. He needs one of the cups and measures the paper. Based on the diagram of the uncurled cup, find the diameter of the cone.
    \[ d = 2 \text{ in} \]

Practice C

Surface Area of Pyramids and Cones

Use the figure for Exercises 1-3. The figure shown can be cut into an open-top cone.
1. The radius of the top of the cone.
2. Find the radius of a circle that has the same area as the lateral area of the cone.
3. Name the radius(s) of each circle that has the same area as the lateral area of the cone described in Exercises 4-6. Give exact answers.
4. \[ r = 4 \text{ cm}; r = 1 \text{ in} \]
5. \[ r = 6 \text{ cm}; r = 1 \text{ in} \]
6. \[ r = 8 \text{ cm}; r = 2 \text{ in} \]

7. A cone with an open base can be formed from any partial circle. Develop a formula for the lateral area of a cone based on \( r \), the slant height, and \( \theta \), the degree measure of the interior angle of the partial circle. (For instance, for the figure shown \( \theta = 90^\circ \).)
   \[ L = \frac{2 \pi r \theta}{360} \]

The figures below can be cut into open frustra. Find the lateral area of the outside and the radius of both bases in each frustrum. Round to the nearest tenth.

8. \[ L = 16.8 \text{ in}^2; r_1 = 0.3 \text{ in}; r_2 = 1 \text{ in} \]
9. \[ L = 17.3 \text{ m}^2; r_1 = 2.4 \text{ m}; r_2 = 3.1 \text{ m} \]
10. Tony is driving through the desert when he notices the gas is low on oil. Sue has a few quarts of motor oil in the front of the car, but she does not have a funnel. Fortunately, Tony finds a piece of 11-by-11-inch notebook paper in the backseat. He tears out the paper to make a funnel with a 1-inch diameter hole in the bottom and the longest slant height possible. Find the diameter of the top of the funnel. (Use the pattern Tony will cut out before cutting up the funnel.)
    \[ \frac{5}{2} \text{ in} \]

Retake

Surface Area of Pyramids and Cones

Lateral and Surface Area of a Regular Pyramid

Lateral Area
- The lateral area of a regular pyramid with perimeter \( P \) and slant height \( s \).
  \[ L = \frac{1}{2} \text{ph} \]
- The surface area of a regular pyramid with lateral area \( L \) and base area \( B \).
  \[ S = L + B \]

Lateral and Surface Area of a Right Cone

Lateral Area
- The lateral area of a right cone with radius \( r \) and slant height \( s \).
  \[ L = \pi rs \]
- The surface area of a right cone with lateral area \( L \) and base area \( B \).
  \[ S = L + B \]

Find the lateral area and surface area of each regular pyramid. Round to the nearest tenth.

1. \[ L = 90 \text{ ft}^2; S = 115 \text{ ft}^2 \]
2. \[ L = 36 \text{ m}^2; S = 45.4 \text{ m}^2 \]

Find the lateral area and surface area of each right cone. Give your answers in terms of \( \pi \).

3. \[ L = 24 \pi \text{ in}^2; S = 33 \pi \text{ in}^2 \]
4. \[ L = 90 \pi \text{ cm}^2; S = 129 \pi \text{ cm}^2 \]
Surface Area of Pyramids and Cones

The radius and slant height of the cone at right are doubled. Describe the effect on the surface area.

Original surface area: \[ S = \pi r^2 + \pi r l \]

New surface area, dimensions doubled: \[ S = \pi (2r)^2 + \pi (2r)(2l) \]

\[ S = 4\pi r^2 + 4\pi rl \]

If the dimensions are doubled, then the surface area is multiplied by 4.

Describe the effect of each change on the surface area of the given figure.

1. The dimensions are tripled.
2. The dimensions are multiplied by \( \frac{1}{2} \).

The surface area is multiplied by 9.

The surface area is multiplied by \( \frac{1}{4} \).

Find the surface area of each composite figure.

1. Hint: Do not include the base area of the pyramid or the upper surface area of the rectangular prism.

\[ S = 133 \text{ in}^2 \]

\[ S = 8\pi \text{ cm}^2 \]

Problem Solving

1. Find the diameter of a right cone with slant height 15 cm and surface area 260 cm². Find the base radius of the cone.

16 cm

2. Find the surface area of a regular pentagonal pyramid with base area 40 in² square meters and slant height 13 in. Round to the nearest tenth.

222.4 mm²

3. A piece of paper in the shape shown is folded to form a cone. What is the diameter of the base of the cone that is formed? Round to the nearest tenth.

18.7 in.

Choose the best answer.

4. A square pyramid has a base with a side length of 6 cm and a slant height of 10 cm. If its 4 congruent faces are each multiplied by \( \frac{3}{4} \), what will be the new surface area of the pyramid?

\[ 72 \text{ cm}^2 \]

5. A cone has a base diameter of 6 cm. What is the slant height of the cone if its height is 10 cm?

5 cm

6. A cone has a base diameter of 6 yards. What is the slant height of the cone if its height is 10 yards?

8.66 yd

7. A cone has a base diameter of 6 yards. What is the slant height of the cone if its height is 10 yards?

8.66 yd

Challenge

1. A cone has a base diameter of 6 cm. What is the slant height of the cone if its height is 10 cm?

5 cm

2. A cone has a base diameter of 6 yds. What is the slant height of the cone if its height is 10 yds?

8.66 yd

3. The lateral area of a cone is 40 \( \pi \) cm². If the radius of the base is 5 cm, what is the height of the cone?

10 cm

4. The lateral area of a cone is 40 \( \pi \) cm². If the radius of the base is 5 cm, what is the height of the cone?

10 cm

Reading Strategies

Compare and Contrast

The diagram below summarizes the similarities and differences between regular pyramids and right cones and their lateral and surface areas.

<table>
<thead>
<tr>
<th>Pyramid</th>
<th>Cone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both have a vertex.</td>
<td>Both have a slant height.</td>
</tr>
<tr>
<td>Both have a base.</td>
<td>Both have a base.</td>
</tr>
<tr>
<td>Lateral area = perimeter ( \times ) height</td>
<td>Lateral area = ( \pi ) ( r ) ( l )</td>
</tr>
<tr>
<td>Surface area = base area + lateral area</td>
<td>Surface area = ( \pi ) ( r ) ( l ) + ( \pi ) ( r ) ( r )</td>
</tr>
</tbody>
</table>

Answer the following.

1. Look at the pyramid and cone above. Why do you think the slant height is so named?

Possible answer: It is perpendicular to the base, is slanted diagonally, and the height of a lateral face.

2. Look at the formulas for surface area for each figure. Why do you think the formula for the pyramid uses the area of the base and the formula for the cone does not?

The base of a pyramid could be different polygons. The base of a cone is always a circle.

Find the lateral area and surface area of each figure. Round to the nearest tenth if necessary.

3. \[ L = 5 \pi \text{ cm} \]

4. \[ L = 4 \pi \text{ in} \]

5. \[ S = 30 \pi \text{ cm} \]

6. \[ S = 20 \pi \text{ in} \]